Appendix A: Model psychometric functions

This appendix derives the mathematical expression of the psychometric functions in Eqs. 3 and 5 and lists the variants that hold under alternative conditions. Table A1 lists how the mean $\mu_d$ and the standard deviation $\sigma_d$ of $D$ vary with test size $x$ according to which semicircle was on each side of the screen and which eye was each semicircle seen with. As seen in the table, for any given arrangement of color filters before the eyes, interchanging the positions of the semicircles on the screen leaves $\sigma_d^2$ unchanged but reverses the sign of $\mu_d$.

Equations 3 in the main paper apply under the conditions in the top half of Table A1. For the conditions in the bottom half, where the red filter is placed before the left eye and the green filter is placed before the right eye, Eqs. 3 become

<table>
<thead>
<tr>
<th>Filter</th>
<th>Position of each color semicircle (color on left ; color on right)</th>
<th>mean, $\mu_d$</th>
<th>variance, $\sigma_d^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OS: green</td>
<td>green ; red</td>
<td>$\mu_{OS}(x_s) - \mu_{OD}(x)$</td>
<td>$\sigma_{OD}^2(x) + \sigma_{OS}^2(x_s)$</td>
</tr>
<tr>
<td>OD: red</td>
<td>red ; green</td>
<td>$\mu_{OD}(x) - \mu_{OS}(x_s)$</td>
<td></td>
</tr>
<tr>
<td>OS: red</td>
<td>green ; red</td>
<td>$\mu_{OD}(x_s) - \mu_{OS}(x)$</td>
<td>$\sigma_{OS}^2(x) + \sigma_{OD}^2(x_s)$</td>
</tr>
<tr>
<td>OD: green</td>
<td>red ; green</td>
<td>$\mu_{OS}(x) - \mu_{OD}(x_s)$</td>
<td></td>
</tr>
</tbody>
</table>
\[ \Psi_L(x) = \Phi \left( \frac{\delta_1 - \left( \mu_{OD}(x) - \mu_{OS}(x) \right)}{\sqrt{\sigma_{OS}(x) + \sigma_{OD}(x)}} \right), \quad (A1a) \]

\[ \Psi_R(x) = \Phi \left( \frac{\delta_1 - \left( \mu_{OS}(x) - \mu_{OD}(x) \right)}{\sqrt{\sigma_{OS}(x) + \sigma_{OD}(x)}} \right), \quad (A1b) \]

\[ \gamma_L(x) = \Phi \left( \frac{\delta_2 - \left( \mu_{OD}(x) - \mu_{OS}(x) \right)}{\sqrt{\sigma_{OS}(x) + \sigma_{OD}(x)}} \right) - \Phi \left( \frac{\delta_1 - \left( \mu_{OD}(x) - \mu_{OS}(x) \right)}{\sqrt{\sigma_{OS}(x) + \sigma_{OD}(x)}} \right), \quad (A1c) \]

\[ \gamma_R(x) = \Phi \left( \frac{\delta_2 - \left( \mu_{OS}(x) - \mu_{OD}(x) \right)}{\sqrt{\sigma_{OS}(x) + \sigma_{OD}(x)}} \right) - \Phi \left( \frac{\delta_1 - \left( \mu_{OS}(x) - \mu_{OD}(x) \right)}{\sqrt{\sigma_{OS}(x) + \sigma_{OD}(x)}} \right), \quad (A1d) \]

and Eqs. 5 become

\[ \frac{\delta_k - \left( \mu_{OS}(x) - \mu_{OD}(x) \right)}{\sqrt{\sigma_{OS}(x) + \sigma_{OD}(x)}} = \delta_k^* + \alpha^* - \beta^* x, \quad (A2a) \]

\[ \frac{\delta_k - \left( \mu_{OD}(x) - \mu_{OS}(x) \right)}{\sqrt{\sigma_{OS}(x) + \sigma_{OD}(x)}} = \delta_k^* - \alpha^* + \beta^* x, \quad (A2b) \]

with \( \alpha^* = (\alpha_{OD} - \alpha_{OS} + \beta_{OD}s) / \sqrt{\kappa_{OD} + \kappa_{OS}}, \) \( \beta^* = \beta_{OS} / \sqrt{\kappa_{OD} + \kappa_{OS}}, \) and \( \delta_k^* = \delta_k / \sqrt{\kappa_{OD} + \kappa_{OS}}. \)

In the full binocular condition, the psychophysical function embodies joint influences from both eyes but it may vary with stimulus color due to the size–color illusion. When the standard stimulus is red, Eqs. 3 become

\[ \Psi_L(x) = \Phi \left( \frac{\delta_1 - \left( \mu_{red}(x) - \mu_{green}(x) \right)}{\sqrt{\sigma_{green}(x) + \sigma_{red}(x)}} \right), \quad (A3a) \]

\[ \Psi_R(x) = \Phi \left( \frac{\delta_1 - \left( \mu_{green}(x) - \mu_{red}(x) \right)}{\sqrt{\sigma_{green}(x) + \sigma_{red}(x)}} \right), \quad (A3b) \]

\[ \gamma_L(x) = \Phi \left( \frac{\delta_2 - \left( \mu_{red}(x) - \mu_{green}(x) \right)}{\sqrt{\sigma_{green}(x) + \sigma_{red}(x)}} \right) - \Phi \left( \frac{\delta_1 - \left( \mu_{red}(x) - \mu_{green}(x) \right)}{\sqrt{\sigma_{green}(x) + \sigma_{red}(x)}} \right), \quad (A3c) \]

\[ \gamma_R(x) = \Phi \left( \frac{\delta_2 - \left( \mu_{green}(x) - \mu_{red}(x) \right)}{\sqrt{\sigma_{green}(x) + \sigma_{red}(x)}} \right) - \Phi \left( \frac{\delta_1 - \left( \mu_{green}(x) - \mu_{red}(x) \right)}{\sqrt{\sigma_{green}(x) + \sigma_{red}(x)}} \right), \quad (A3d) \]
and Eqs. 5 become

\[
\frac{\delta_k - (\mu_{\text{green}}(x) - \mu_{\text{red}}(x_s))}{\sqrt{\sigma^2_{\text{green}}(x) + \sigma^2_{\text{red}}(x_s)}} = \delta^*_k + \alpha^* - \beta^* x, \quad (A4a)
\]

\[
\frac{\delta_k - (\mu_{\text{red}}(x_s) - \mu_{\text{green}}(x))}{\sqrt{\sigma^2_{\text{green}}(x) + \sigma^2_{\text{red}}(x_s)}} = \delta^*_k - \alpha^* + \beta^* x, \quad (A4b)
\]

with \( \alpha^* = (\alpha_{\text{red}} - \alpha_{\text{green}} + \beta_{\text{red}} x_s) / \sqrt{\kappa_{\text{red}} + \kappa_{\text{green}}} \), \( \beta^* = \beta_{\text{green}} / \sqrt{\kappa_{\text{red}} + \kappa_{\text{green}}} \), and \( \delta^*_k = \delta_k / \sqrt{\kappa_{\text{red}} + \kappa_{\text{green}}} \).

Subscripts “red” and “green” are swapped if the standard stimulus is green instead.

**Appendix B: Approximate non-rejection regions for measured PSEs**

A \( P\% \) non-rejection region is the range of values for measured PSEs that can be obtained via some psychophysical procedure on \( P\% \) of the occasions when the true PSE is at some specified location. Bootstrap simulation methods were used to obtain central 95% non-rejection regions around the theoretical PSEs in each of the nine experimental conditions under each viewing mode. Because the precise location of the true PSE only contributes an anchor point, without loss of generality simulations assumed \( \mu_{\text{OD}} = \mu_{\text{OS}} \) and \( \sigma_{\text{OD}} = \sigma_{\text{OS}} \) so that the true PSE is at the standard level. Simulations also assumed \( \alpha_i = \gamma_i = 0 \) in Eqs. 1, as \( \alpha_i \) only sets an arbitrary and inconsequential anchor in the subjective continuum (see García-Pérez, 2014) whereas \( \gamma_i \) produces negligible effects when the range of stimulus levels is relatively narrow. This leaves \( \beta_{\text{OS}} = \beta_{\text{OD}} = \beta \) and \( \kappa_{\text{OS}} = \kappa_{\text{OD}} = \kappa \) as the only relevant sensory parameters.

Parameter values needed to generate the data (\( \beta, \kappa, \delta_1 \), and \( \delta_2 \)) varied within ranges that were consistent with the spread of the psychometric functions observed in our empirical data and with the patterns of usage of the “can’t tell” response option also observed in our empirical data. Regarding the spread of psychometric functions, three different scenarios were considered: broad, narrow, and assorted. In the “broad-spread” scenario, \( \beta \) was uniformly distributed on [0.6, 0.8] and \( \kappa \) was uniformly distributed on [1.2, 1.4]; in the “narrow-spread” scenario, \( \beta \) was uniformly distributed on [1.2, 1.4] and \( \kappa \) was uniformly distributed on [0.6, 0.8]; in the “assorted” scenario, \( \beta \) was uniformly distributed on [0.6, 1.4] and \( \kappa \) was uniformly distributed on [0.6, 1.4]. Regarding patterns of usage of the “can’t tell” response option, decision boundaries across replicates were drawn so that their midpoint \( (\delta_1 + \delta_2)/2 \) was
uniformly distributed on $[-2.5\sqrt{\kappa}, 2.5\sqrt{\kappa}]$ and their span $\delta_2 - \delta_1$ was uniformly distributed on $[0, 5\sqrt{2\kappa}]$, using the value for $\kappa$ drawn for the corresponding replicate.

In each scenario, data were generated for 10,000 replicates undergoing the psychophysical procedure used in the free viewing mode (amounting to 192 trials per replicate, as described in Method) and, separately, the psychophysical procedure used in the short presentation mode (amounting to 384 trials per replicate, as described also in Method). Data from each replicate in each case were used to obtain parameter estimates as was done with empirical data from real observers and the measured PSE was analogously obtained. The outcome measure for these analyses is the relative error of estimation, defined as the difference between true PSE and measured PSE scaled according to the estimated spread of the psychometric functions. For each replicate, parameter estimates were used to compute the relative error of estimation, which is thus $(\alpha^* / \beta^* - x_\lambda) \beta^* = \alpha^* - \beta^* x_\lambda$. The 2.5-th and the 97.5-th percentiles of the distribution of relative errors of estimation across replicates were then determined, which define the 95% non-rejection region around the true PSE.

Figure B1 shows the distribution of relative errors of estimation in each scenario (rows) under each psychophysical procedure (columns), with the boundaries of the non-rejection region indicated by the
short vertical lines and the numerals next to them. The width of these regions varies across viewing conditions (compare left and right panels in each row), a natural consequence of the fact that the short presentation mode gathered data from a larger number of trials and, thus, allowed more accurate parameter estimates. Because of the normalization with respect to the spread of the estimated psychometric functions, the width of the distributions is relatively unaffected by large differences in the range of true spreads of the underlying psychometric functions in each scenario. We will use 95% non-rejection regions from the “assorted” scenario (bottom row in Fig. B1) to judge the significance of deviations between measured and expected PSEs in our empirical study. Thus, in the free viewing mode, measured PSEs (estimated as $x_{\text{PSE}} = \alpha' / \beta'$, as described in Method) are regarded as significantly different from the expected PSE at $x_{\text{exp}}$ if they lie outside the interval $[x_{\text{exp}} - 0.36 / \beta', x_{\text{exp}} + 0.37 / \beta']$; similarly, in the short presentation mode, measured PSEs are regarded as significantly different from expected PSEs if they lie outside the interval $[x_{\text{exp}} - 0.25 / \beta', x_{\text{exp}} + 0.25 / \beta']$. Individual non-rejection regions are computed using the estimate of $\beta'$ for each observer in each condition and they are shown as gray areas around the theoretical PSE in plots of data and fitted psychometric functions (Figs. 5–7).

**Appendix C: Additional conditions**

This appendix presents and discusses the results obtained in two additional conditions that test speculations about the origin of some side issues that arose from our main experiments.

The first issue is that, in the free viewing mode, patterns of strong underestimation of lens-induced aniseikonia at negative magnifications broke at nominal −4% for some observers (see the results for observers #1, #2, and #6 in the left panel of Fig. 8 in the paper). This is presumably caused by the relatively small retinal size involved in this study: Even when alignment at the foveated extreme might be apparent, the shorter-than-standard length of the test at the other extreme is also very conspicuous in near peripheral vision and observers may have relied more on peripheral information on size than on foveal information on alignment. An analogous situation occurs in reverse at large positive magnifications, but the effect would be weaker because now the test has a larger-than-standard size and its endpoint at the other extreme falls farther into the periphery. The tenability of this surmise was checked out using a larger configuration (721 pixels for the standard diameter) so as to match the
retinal size of the (rectangular) standard stimulus in Fullard et al.’s (2007) study with the AI Version 2. With an angular subtense of 15.93 deg, the poorer (farther) peripheral information may not counter the evidence of alignment gathered foveally. The inconclusive results are shown in Fig. C1.

Underestimation at large negative magnifications was indeed stronger for two observers (#1 and #2). This suggests that judgments of size based on alignment information gathered foveally and undisturbed by contradictory but distant peripheral evidence are relatively immune to the effects of size lenses, resulting in a stronger underestimation of lens-induced aniseikonia. For observer #6, on the other hand, the use of larger stimuli did not produce any discernible effect. This could result from a failure of the larger stimuli to reach sufficiently peripheral regions for this observer, but also from a failure to comply with the instructions to judge size by checking for alignment at the top and at the bottom, as these data were collected after the observer had completed all of the short presentation conditions with instructions to maintain central fixation.

The second issue of concern is the relatively strong asymmetry displayed by observers #3, #6, and #9 in the short presentation mode under the two haploscopic conditions without size lenses but color filters swapped between the eyes (right half of Fig. 6 in the paper). With the green and red filters respectively placed before the left and right eyes, data showed what appears to be a mild natural aniseikonia for observers #3 and #6 and a relatively stronger natural aniseikonia in the opposite direction for observer #9 (see their data in the right panel of Fig. 8 in the paper). Recall from Fig. 5 in the paper that, in the short presentation mode, observers #3 and #9 did not show any traces of a size–color illusion and that observer #6 only showed weak traces of it. Then, if this were actually natural aniseikonia, the shift of the data paths for these observers with respect to the diagonal should turn into a shift by the same amount but in the opposite direction when color filters are swapped.

Fig. C1. Relation between expected and measured aniseikonia under the free viewing mode with large stimuli.
between the eyes. This symmetric shift was not observed in the only condition (without size lenses) in which color filters had been swapped between the eyes (see the results in this condition for observers #3, #6, and #9 in the right half of Fig. 6 in the paper). Additional data were thus collected for these observers with color filters swapped between the eyes and size lenses placed before the left eye instead. The results are shown in Fig. C2. Data now lie virtually on the diagonal for observers #3 and #6 and far below the diagonal for observer #10. The asymmetry persists, attesting to unidentified aspects of color perception contaminating aniseikonia measurements.

Fig. C2. Relation between expected and measured aniseikonia with each possible placement of color filters before the eyes under the short presentation mode. For ease of comparison, solid symbols and continuous lines replot data from the right panel of Fig. 8 in the paper for the observers of concern. Open symbols and dotted lines plot data for the same observers when color filters were swapped between eyes (red before the left eye and green before the right eye).

References
