Supplemental Material: Interpretation of Regression Equations for Figure 5

When estimating person and item measures, the Rasch model normalizes the measures to the square root of the error variance associated with the items (*i.e.*, dry eye indicators). Let us define S_n to be the "true" dry eye severity of person n. As implied by the distribution of infit mean squares for the indicators, the error variance for signs, σ_i^2, is greater than the error variance for symptoms, σ_y^2. With normalization, the estimated dry eye severity for person n from only indicator signs is

$$\tilde{\tilde{S}}_{ni} = \frac{S_n}{\sigma_i} + b_i, \qquad (1)$$

with b_i being a constant that is specific to the subset of signs used as indicators (because the origin of the interval measurement scale is chosen to be the average of the item measures). Similarly, the estimated dry eye severity for person n from only indicator symptoms is

$$\tilde{\tilde{S}}_{ny} = \frac{S_n}{\sigma_y} + b_y, \qquad (2)$$

which is the same as eq. (1), but with appropriate notational changes in the variance and the intercept. Finally, when dry eye severity is estimated from both indicator signs and symptoms, the estimated severity measure is specified as

$$\tilde{\tilde{S}}_{niy} = \frac{S_n}{\sigma_{iy}} + b_{iy}, \qquad (3)$$

where σ_{iy} refers to the combined error variances of signs and symptoms. Since all three equations are linear functions of S_n, *e.g.*,

$$S_n = \sigma_{iy} \tilde{\tilde{S}}_{niy} - \sigma_{iy} b_{iy}, \qquad (4)$$

If all three estimated variables are just rescalings of the same variable, then the two measures estimated from subsets of the indicators can be expressed as linear functions of the measure estimated from all of the indicators, *i.e.*,

$$\widetilde{S}_{ni} = \frac{\sigma_{iy}}{\sigma_i}\widetilde{S}_{niy} + c_i, \tag{5}$$

where $c_i = b_i - \frac{\sigma_{iy}}{\sigma_i}b_{iy}$, and

$$\widetilde{S}_{ny} = \frac{\sigma_{iy}}{\sigma_y}\widetilde{S}_{niy} + c_y, \tag{6}$$

where $c_y = b_y - \frac{\sigma_{iy}}{\sigma_y}b_{iy}$. These two linear equations, which describe the two scatter plots in figure 6, show that the slopes of the regression lines correspond to the ratios of the error standard deviations. The ratio of the two regression line slopes then can be interpreted as

$$\frac{\sigma_{iy}}{\sigma_y}\Bigg/\frac{\sigma_{iy}}{\sigma_i} = \frac{\sigma_i}{\sigma_y}$$

which, when squared, is the ratio of the sign to symptom error variances.